Fermi National Accelerator Laboratory
PPD / Mechanical Support / Engineering Analysis

Carbon Foil Temperature

Zhijing Tang
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SUMMARY New development in proton driver study calls for thicker stripping foil (previous thickness of the carbon foil is 300 μg/cm^2, new thickness is 400 to 600 μg/cm^2). There are some concerns about weather thicker foil will work or not. This analysis is to look at the temperature of the carbon foil. The results show that the maximum temperature increase is about 50 K, and there is no temperature build-up in the foil.

Beam Pulse

Weiren Chou provided following beam parameters:

Beam energy = 8 GeV
Beam current = 25 mA (8 mA)
Pulse duration = 1 ms (3 ms)
Repeat rate = 1/1.5 Hz
Beam size = 4mmx8mm (Gaussian)

Electron charge e = 1.6e-19 C. Beam pulse has (25e-3)(1e-3) = 25e-6 C. One beam pulse has N = 25e-6 / 1.6e-19 = 15.6e13 protons.

The distribution of the beam particles over the cross section is assumed Gaussian

$$P = \frac{N}{2\pi\sigma_x\sigma_y} \; e^{-x^2/2\sigma_x^2} \; e^{-y^2/2\sigma_y^2}$$

From the beam size given, we let σ_x = 0.2 cm, and σ_y = 0.1 cm.

Carbon Foil

From *The Proton Driver Design Study* (FermiLab-TM-2136) section 11.3, we have following data for carbon foil:

Density ρ = 2.0 g/cm^3
Specific heat c = 0.165 cal/g-K = 0.69 J/g-K
Thermal conductivity k = 0.057 cal/cm-K-s = 0.24 w/cm-K
Emissivity ε = 0.80

The thickness of the carbon foil is given in μg/cm^2, it should be divided by the density to get thickness in cm. For a 400 μg/cm^2 foil, the thickness is δz = 2 μm.

Energy Deposition

Assume that the energy deposition is instantaneous compared with thermal process, then energy deposition density per pulse is given by following equation

$$S = \frac{N}{2\pi\sigma_x\sigma_y}\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| e^{-x^2/2\sigma_x^2}\ e^{-y^2/2\sigma_y^2}$$

Weiren Chou gives $|\mathrm{d}E/\mathrm{d}z|$ = 1.8 MeV/(g/cm^2). For thickness of 400 μg/cm^2, the energy deposition is (1.8)(400e-6) = 7.2e-4 MeV per particle. The total energy deposition is therefore $N\,|\mathrm{d}E/\mathrm{d}z|\ \delta z$ = (15.6e13)(0.72e-7) = 11.23e6 GeV = 18e-4 J.

Let $A = N\,|\mathrm{d}E/\mathrm{d}z|$ = (18e-4 J)/(2e-4 cm) = 9 J/cm. Energy deposition equation can be written as

$$S = \frac{A}{2\pi\sigma_x\sigma_y}\ e^{-x^2/2\sigma_x^2}\ e^{-y^2/2\sigma_y^2}$$

Temperature Increase

Temperature increase after one pulse of beam is

$$\Delta T = \frac{A/\rho c}{2\pi\sigma_x\sigma_y}\ e^{-x^2/2\sigma_x^2}\ e^{-y^2/2\sigma_y^2}$$

The maximum temperature increase occurs at the center

$$\Delta T_{\max} = \frac{A/\rho c}{2\pi\sigma_x\sigma_y} = 52\text{ K}.$$

Thermal Radiation

If thermal conduction is neglected, the equation of temperature due to radiation can be written as

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{2\varepsilon\sigma}{\delta z\,\rho\,c}\left(T^4 - T_0^4\right).$$

Where the factor 2 is included, to reflect the two radiation surfaces. T_0 = 275 K is the ambient temperature. The Stefan-Boltzmann constant is σ = 5.67e-8 w/m^2-K^4 = 5.67e-12 w/cm^2-K^4. Integration of above equation gives

$$F = -B(t - t_0) = \frac{1}{2}\ln\left(\frac{T - T_0}{T + T_0}\right) + \tan^{-1}\left(\frac{T}{T_0}\right)$$

Where t_0 is the constant of integration, and constant $B = \frac{4\varepsilon\sigma T_0^3}{\delta z\,\rho\,c}$. For δz = 2e-4 cm and T_0 = 275 K, B = 1.3672 s^{-1}. Figure 1 plotted above solution for three thickness cases. It is seen that there is not much temperature accumulation.

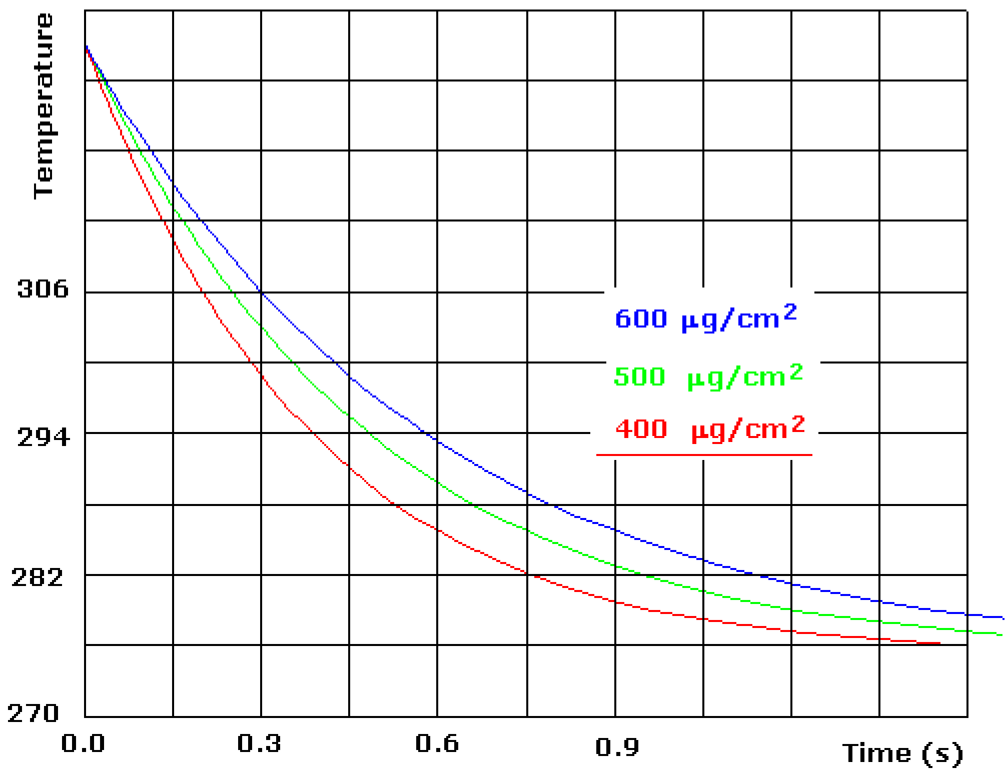


Fig. 1 Temperature Change due to Thermal Radiation

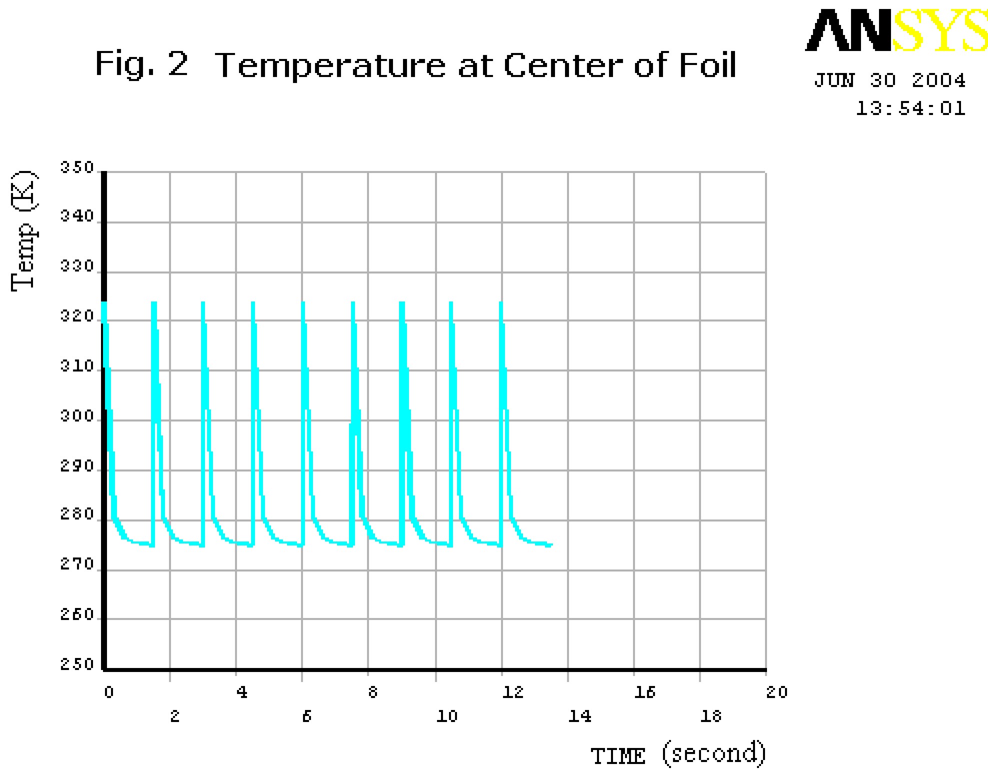
Finite Element Model

Finite element model is built simulate the thermal process. Solid element is used to model the carbon foil, and surface element is used to implement the radiation. In finite element, both conduction and radiation are included. But as expected, the conduction does not show much of the effect. Figures 2 and 3 are for carbon foil with thickness of 400 µg/cm^2, and Figures 4 and 5 are for carbon foil with thickness of 600 µg/cm^2. Figures 2 and 4 show temperature at center changes with time. Figures 3 and 5 show temperature over entire foil at several times. The difference is small.

Conclusion

This analysis shows that the maximum temperature increase is about 50 K, and there is no temperature build-up in the foil.

Fig. 2 Temperature at Center of Foil



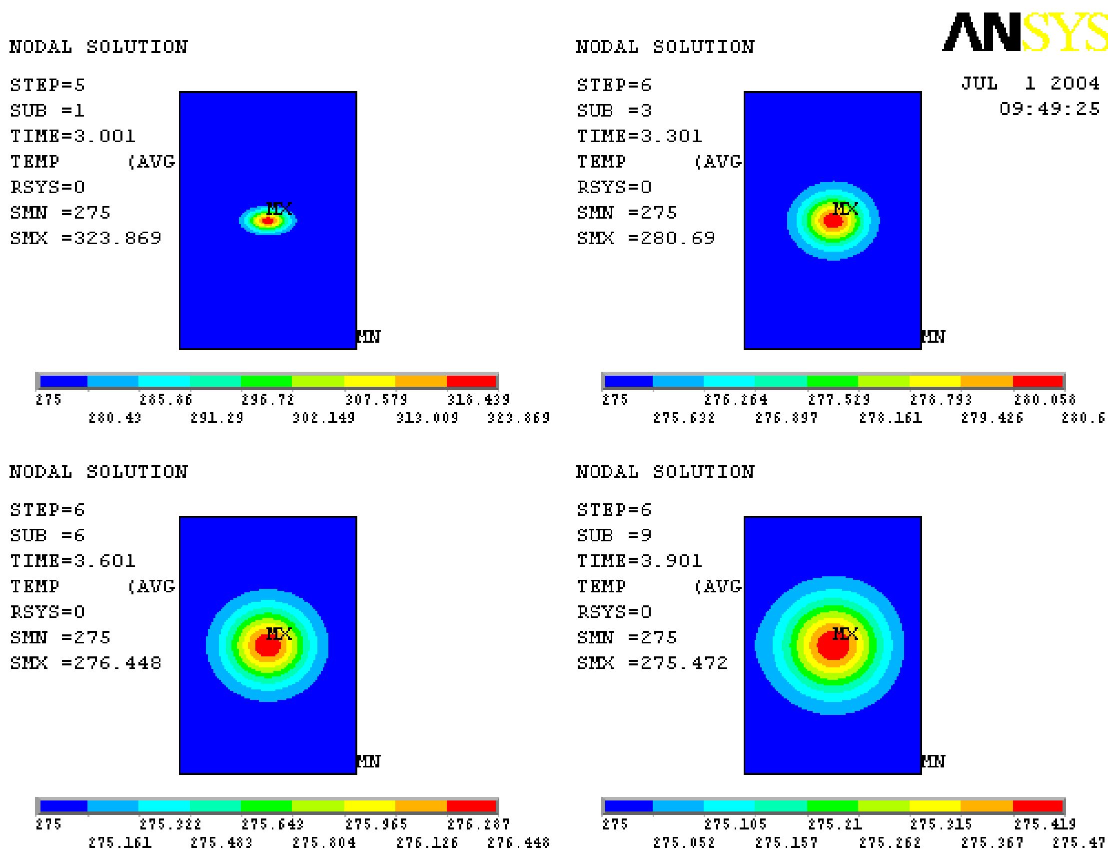
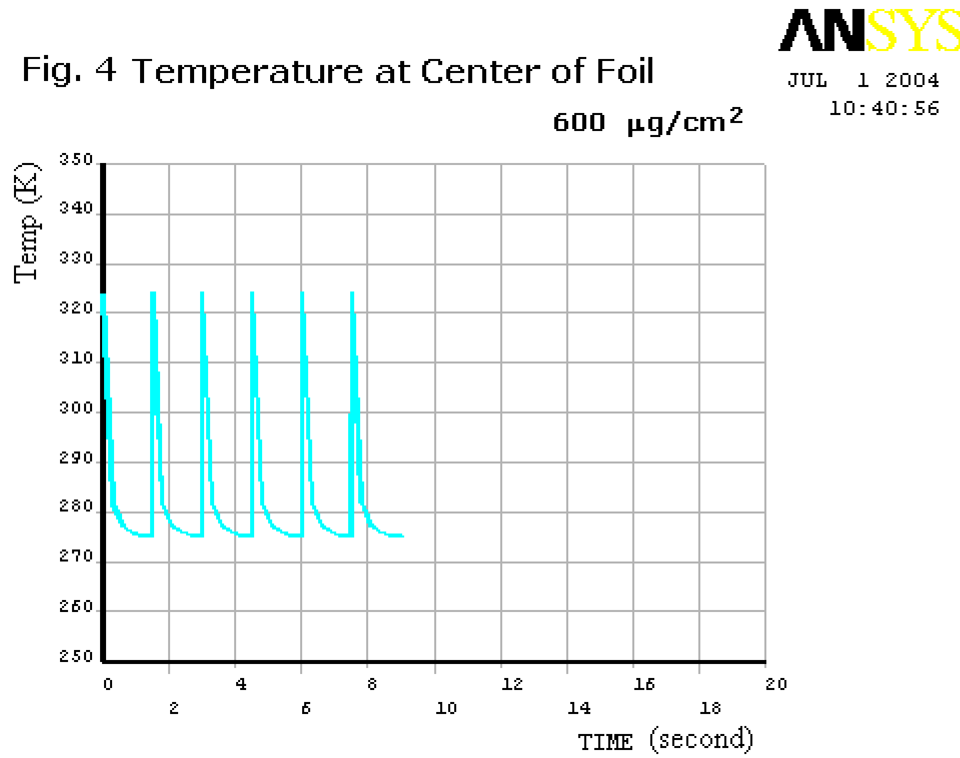


Fig. 3 Foil Temperature at Different Times

Fig. 4 Temperature at Center of Foil



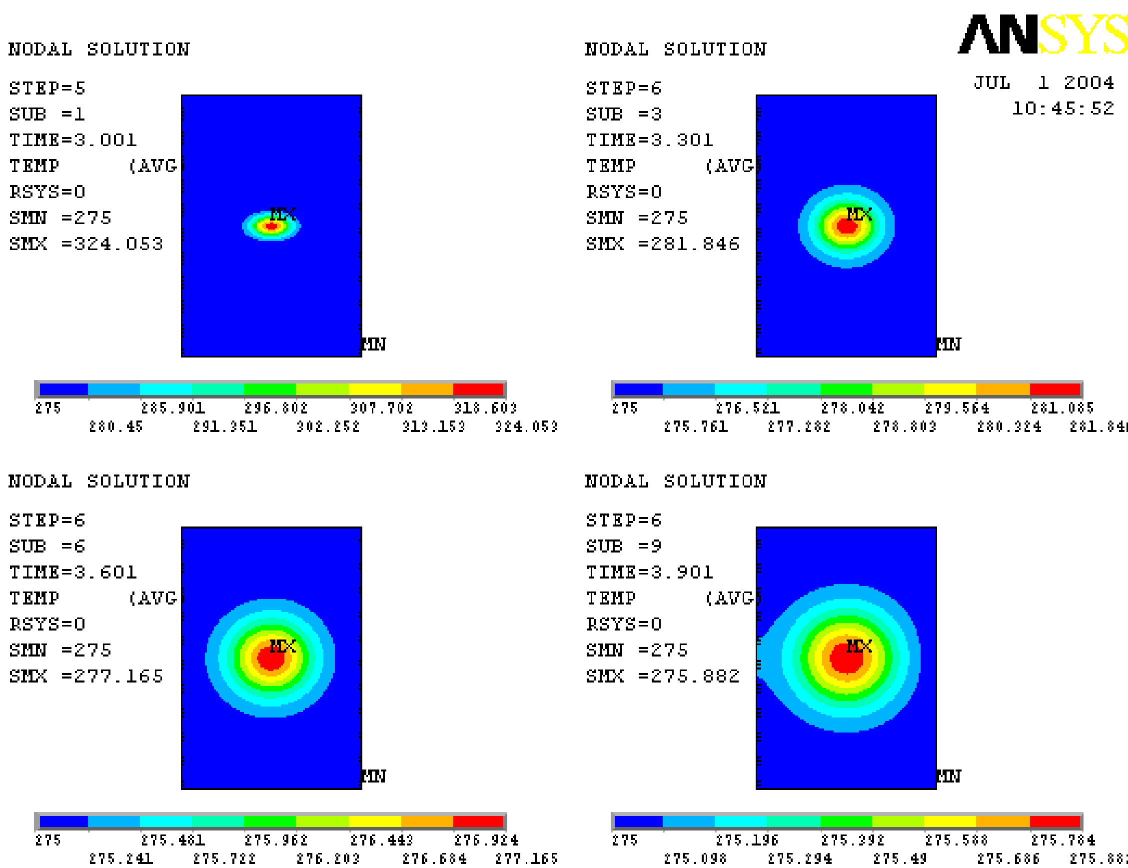


Fig. 5 Foil Temperature at Different Times 600 μg/cm²